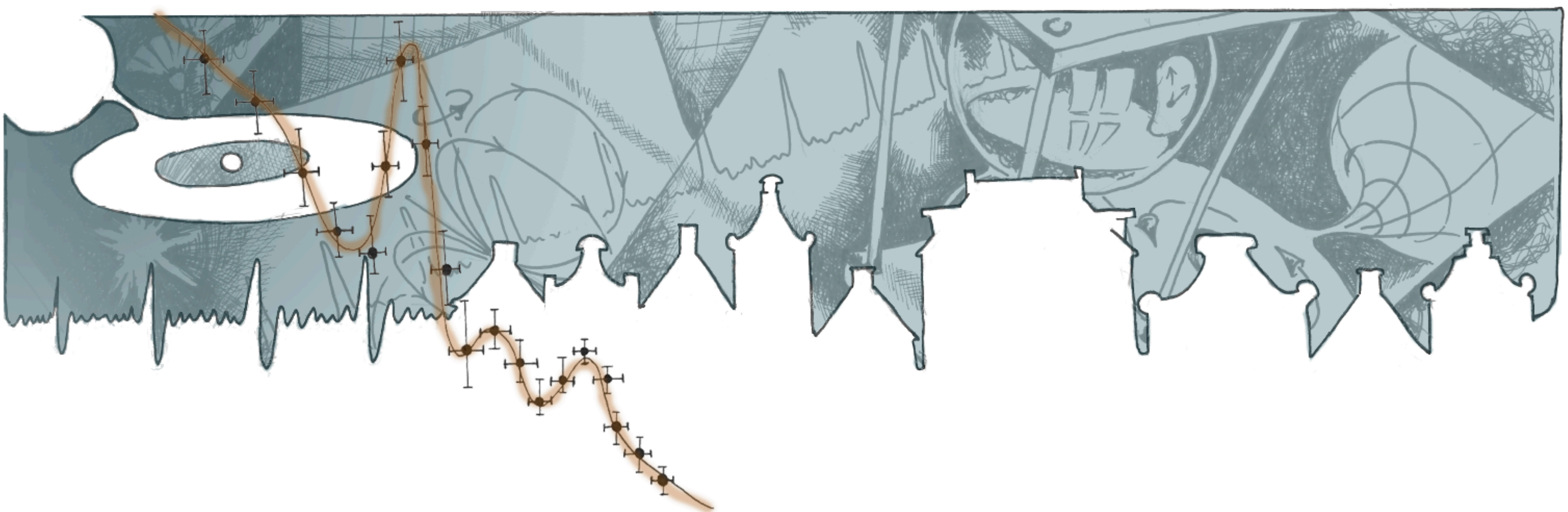

Propagation of mass accretion rate fluctuations in X-ray binaries under influence of viscous diffusion

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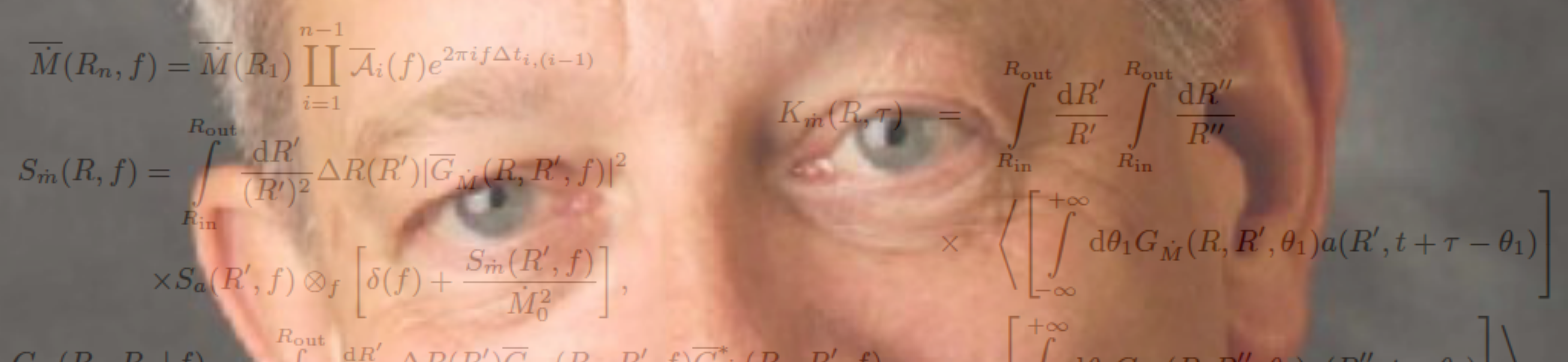


$$\overline{\dot M}(R_n,f)=\overline{\dot M}(R_1)\prod_{i=1}^{n-1}\overline{\mathcal{A}}_i(f)e^{2\pi i f\Delta t_{i,(i-1)}}$$

$$S_{\dot m}(R,f)=\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R'}{(R')^2}\Delta R(R')|\overline{G}_{\dot M}(R,R',f)|^2 \times S_a(R',f)\otimes_f\left[\delta(f)+\frac{S_{\dot m}(R',f)}{\dot M_0^2}\right],$$

$$K_{\dot m}(R,\tau)=\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R'}{R'}\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R''}{R''} \times\left\langle\left[\int_{-\infty}^{+\infty}{\rm d}\theta_1 G_{\dot M}(R,R',\theta_1)a(R',t+\tau-\theta_1)\right] \times\left[\int_{-\infty}^{+\infty}{\rm d}\theta_2 G_{\dot M}(R,R'',\theta_2)a(R'',t-\theta_2)\right]\right\rangle_t$$

$$=\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R'}{R'}\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R''}{R''}\int_{-\infty}^{+\infty}{\rm d}\theta_1\int_{-\infty}^{+\infty}{\rm d}\theta_2 \times G_{\dot M}(R,R',\theta_1)G_{\dot M}(R,R'',\theta_2) \times\langle a(R',t+\tau-\theta_1)a(R'',t-\theta_2)\rangle_t .$$

$$C_{\dot m}(R_1,R_2\,|\,f)=\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R'}{(R')^2}\Delta R(R')\overline{G}_{\dot M}(R_1,R',f)\overline{G}^*_{\dot M}(R_2,R',f) \times S_a(R',f)\otimes_f\left[\delta(f)+\frac{S_{\dot m}(R',f)}{\dot M_0^2}\right], \qquad (34)$$

$$\overline{\dot m}_N = \sum_{i=1}^{N-1}\frac{\Delta R_i}{R_i}\overline{G}_{Ni}\overline{a}_i +\xi\sum_{j=1}^{N-2}\sum_{i=j+1}^{N-1}\frac{\Delta R_i}{R_i}\frac{\Delta R_j}{R_j}\overline{G}_{Ni}\overline{a}_i\otimes_f\overline{G}_{ij}\overline{a}_j +\xi^2\sum_{j=1}^{N-3}\sum_{i=j+1}^{N-2}\sum_{k=i+1}^{N-1}\frac{\Delta R_i}{R_i}\frac{\Delta R_j}{R_j}\frac{\Delta R_k}{R_k} \times\overline{G}_{Nk}\overline{a}_k\otimes_f\overline{G}_{ki}\overline{a}_i\otimes_f\overline{G}_{ij}\overline{a}_j+(\dots),$$

$$v_{\rm r}\simeq\alpha v_\varphi\left(\frac{\rm H}{\rm R}\right)^2$$

$$C_{\dot m}(R_1,R_2\,|\,f)=\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R'}{R'}\int_{R_{\rm in}}^{R_{\rm out}}\frac{{\rm d}R''}{R''}\int_{-\infty}^{+\infty}{\rm d}\theta_1\int_{-\infty}^{+\infty}{\rm d}\theta_2\int_{-\infty}^{+\infty}{\rm d}z \times G_{\dot M}(R_1,R',\theta_1)G_{\dot M}(R_2,R'',\theta_2) \times\gamma_a(R_1,R_2\,|\,z)e^{-2\pi i f(z+\theta_1-\theta_2)}.$$

$$\frac{\partial[x^l\,V(k_i\,x,k_i\,x_{\rm in})]}{\partial x^{2l}}\Big|_{x=x_{\rm out}}=0$$

$$G_F(x,x_1,t)= =2\,x^l\,x_1^{1-l}\,x_{\rm out}^{-2} \times\sum_i\exp\left(-\frac{k_i^2}{8\,l^2}\,\frac{t}{t_{\rm v}}\right)\frac{V(k_i\,x_1,k_i\,x_{\rm in})\,V(k_i\,x,k_i\,x_{\rm in})}{V^2(k_i\,x_{\rm out},k_i\,x_{\rm in})}$$

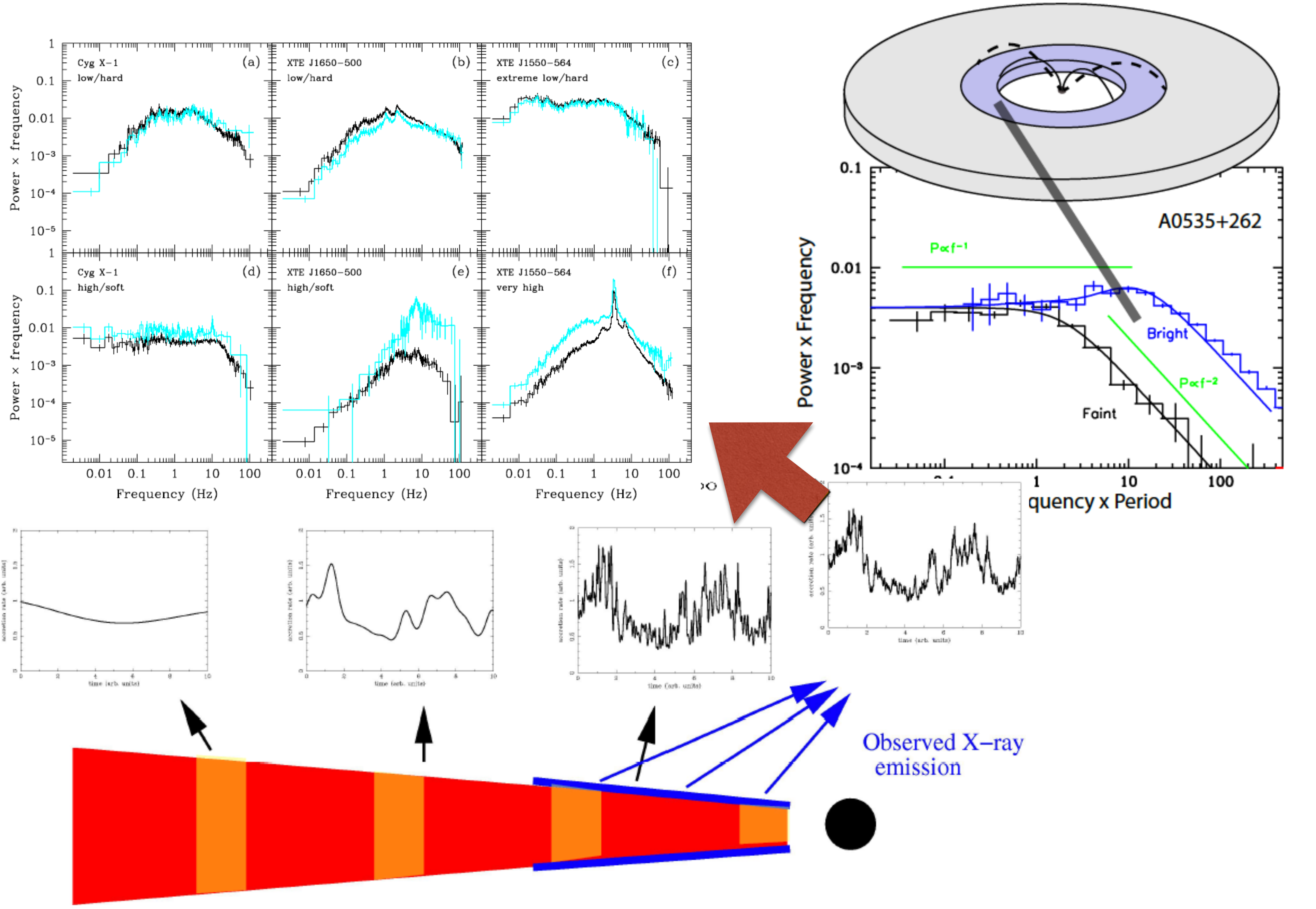
$$(k_ix_{\rm out})J_{-l}(k_ix_{\rm in})-J_{-l-1}(k_ix \dots \quad -\frac{2l}{\dots}$$

$$\int_{E_a^{(1)}}^{E_a^{(2)}}{\rm d}E\,g_3(R,\varphi,E) =\left(-\frac{E^4}{\Phi}\frac{e^{\Phi E}}{e^{\Phi E}-1}+\frac{24}{\Phi^5}\sum_{m=0}^{3}\frac{(-\Phi E)^m}{m!}{\rm Li}_{4-m}\left(e^{\Phi E}\right)\right)\Big|_{E_a^{(1)}}^{E_a^{(2)}}$$

$$S_F(f,\dot M)\propto D^{-4}m^5\dot M_{17}^{-5/2}\sin^2\psi_d \int_{r_{\rm in}}^{r_{\rm out}}{\rm d}r_1\int_{r_{\rm in}}^{r_{\rm out}}{\rm d}r_2\,g_1(r_1)g_1(r_2)C_{\dot m}(r_1,r_2,f) \int^{2\pi}{\rm d}\varphi_1\int_0^{2\pi}{\rm d}\varphi_2\,g_2(r_1,\varphi_1)g_2(r_2,\varphi_2)e^{2\pi f i[\Delta t(r_1,\varphi_1\dots} \int^{E_a^{(2)}}{\rm d}E_2\,g_3(r_1,\varphi_1,E_1)g_3(r_2\dots$$

Black holes

Neutron stars



Basic assumptions

Geometrically thin and **optically thick** accretion disc

Newtonian potential $$\phi_{\rm N} = -\frac{GM}{R}$$

$$\Omega_{\rm K} = \left(\frac{GM}{R^3}\right)^{1/2} = 11.5\left(\frac{m}{R_8^3}\right)^{1/2} \mathrm{rad\,s^{-1}}$$

$$h_{\rm K} = (GMR)^{1/2}$$

The equation of viscous diffusion:

$$\frac{\partial\Sigma(R,t)}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(3\nu\Sigma R^{1/2}\right)\right]$$

Kinematic viscosity: $\nu = \frac{2}{3}\alpha c_{\rm s} H = \frac{2}{3}\frac{\alpha c_{\rm s}^2}{\Omega_{\rm K}}$

generally non-linear equation

We are lucky if $\nu = \nu_0 (R/R_0)^n$

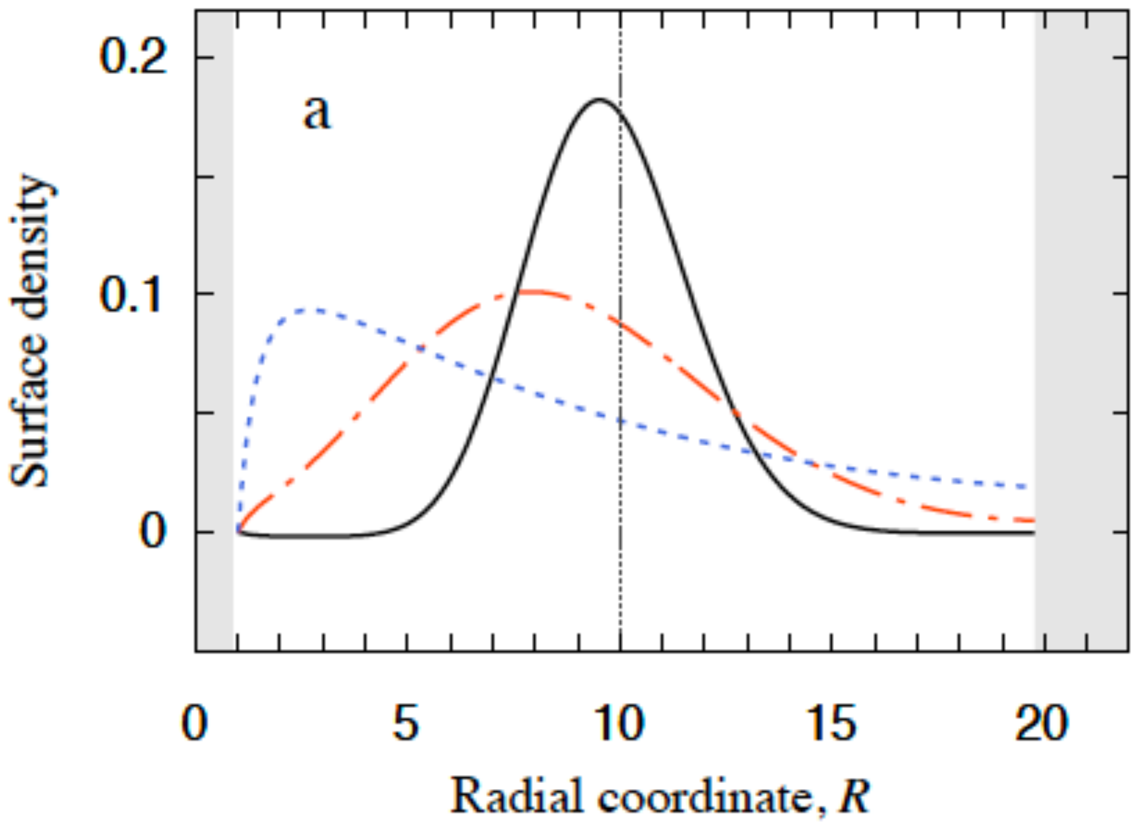
$$\frac{\partial\Sigma}{\partial t} = D_{\rm N}(h,M)\frac{\partial^2(h\nu\Sigma)}{\partial h^2}, \quad D_{\rm N}(h,M) = \frac{3}{4}\frac{(GM)^2}{h^3}$$

linear equation

Green's function

Any solution can be represented as

$$\Sigma(R,t) = \int_{R_{\rm in}}^{R_{\rm out}} G(R,R',t-t_0)\Sigma(R',t_0)\mathrm{d}R'$$



$t = 0.04t_{in}$

$t = 0.16t_{in}$

$t = 0.64t_{in}$

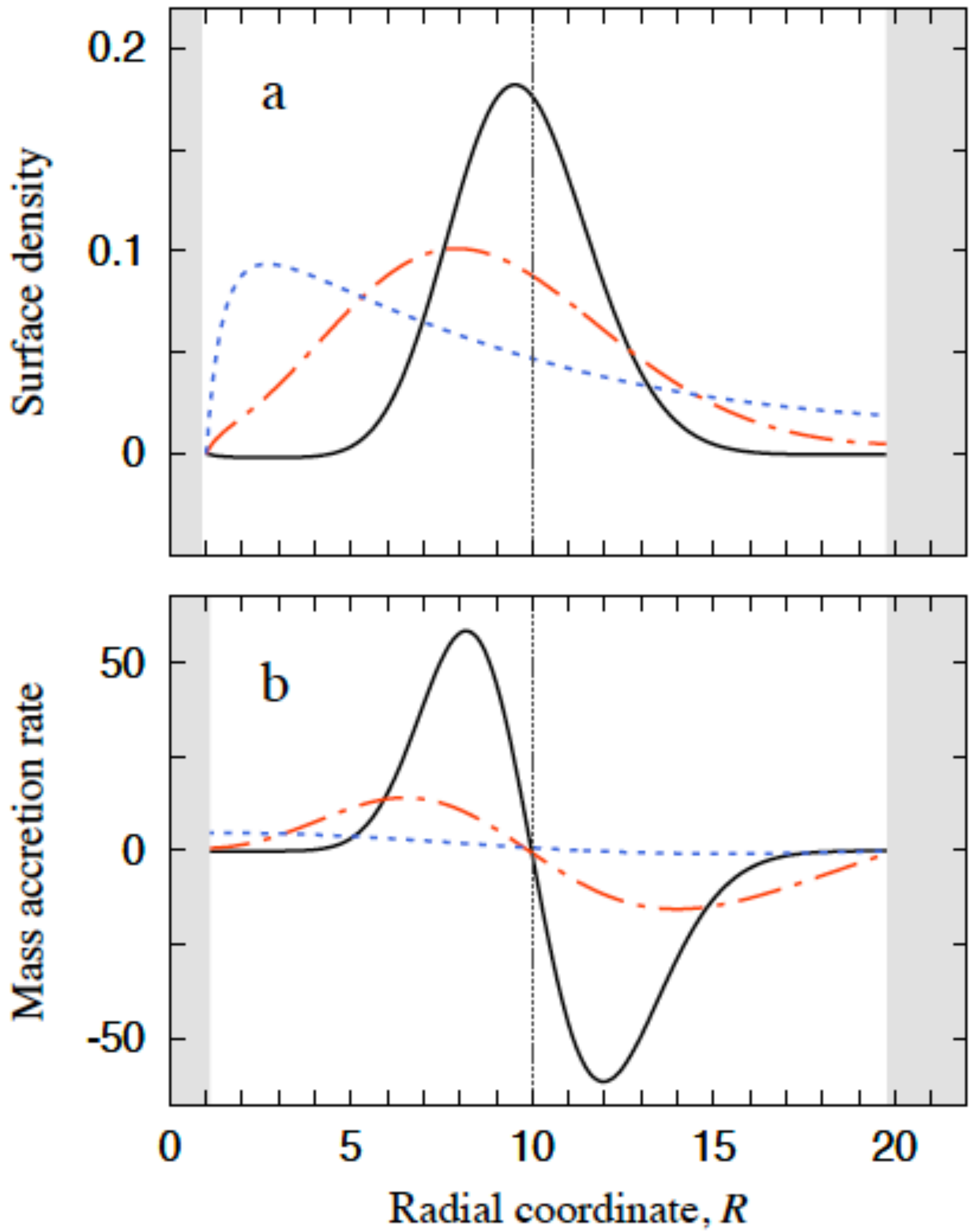
The exact Green's function is defined by viscosity (coefficient n) and boundary conditions.
The exact analytical solutions were found for a few particular cases:

(a) **$R_{in}=0$, $R_{out}=\infty$** *(Lynden-Bell & Pringle, 1974)*
(b) **$R_{in}>0$, $R_{out}=\infty$** *(Tanaka, 2011)*
(c) **$R_{in}=0$, $R_{out}<\infty$** *(Lipunova, 2015)*
(d) **$R_{in}>0$, $R_{out}<\infty$** *(Mushtukov+, 2019)*
(e) **$R_{in}=R_{isco}$, $R_{out}=\infty$, GR Green functions** *(Balbus, 2017)*

Green's function

Any solution can be represented as

$$\Sigma(R,t) = \int_{R_{\rm in}}^{R_{\rm out}} G(R,R',t-t_0)\Sigma(R',t_0)\mathrm{d}R'$$



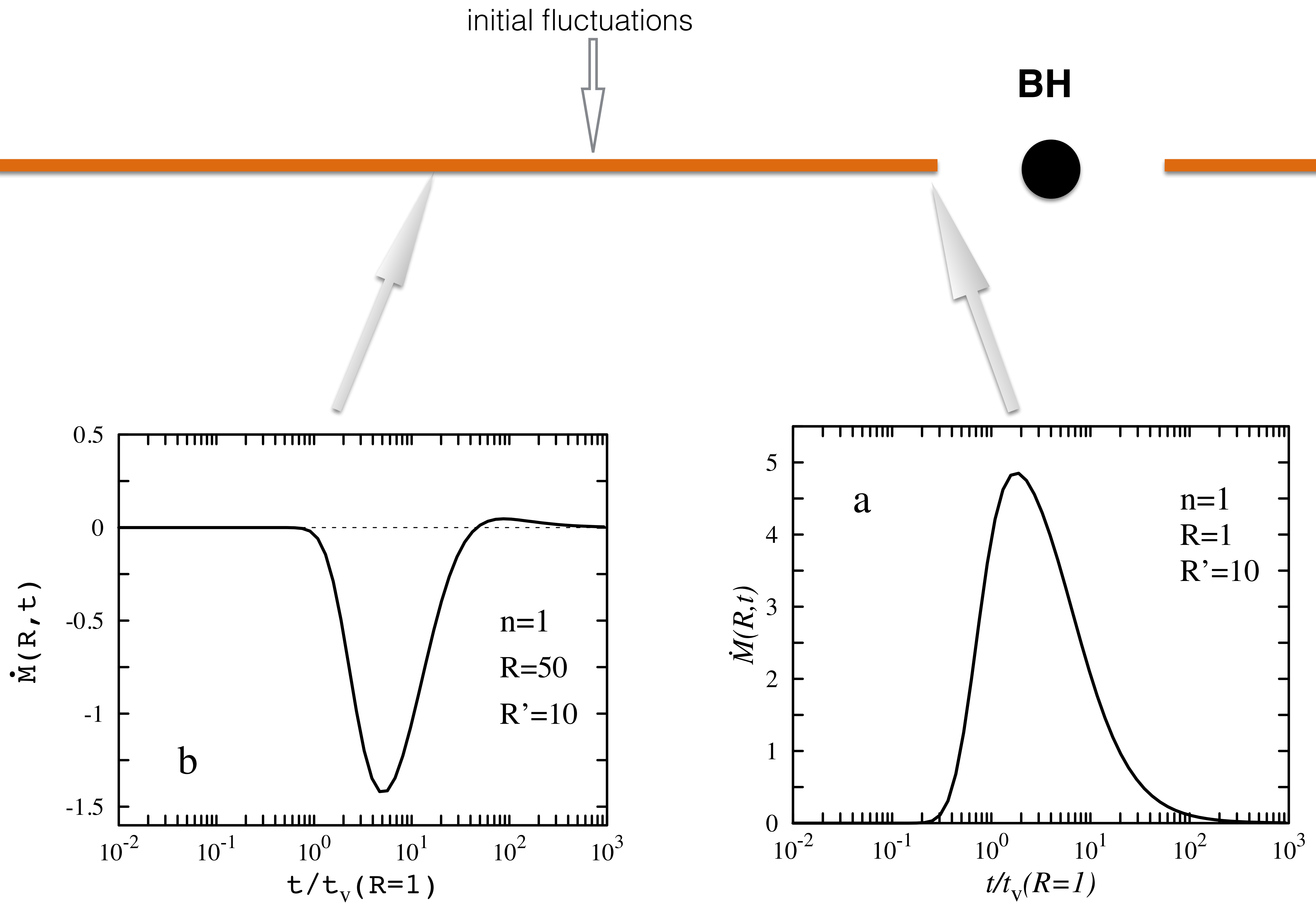
Corresponding mass accretion rate:

$$\dot{M}(R,t) = 6\pi R^{1/2}\frac{\partial}{\partial R}\left(\nu\Sigma(R,t)R^{1/2}\right)$$

Green function for the mass accretion rate:

$$G_{\dot{M}}(R,R',t) = 6\pi R^{1/2}\frac{\partial}{\partial R}\left(\nu G(R,R',t)R^{1/2}\right)$$

Mass accretion rate fluctuations in the time domain



Mass accretion rate fluctuations in the time domain

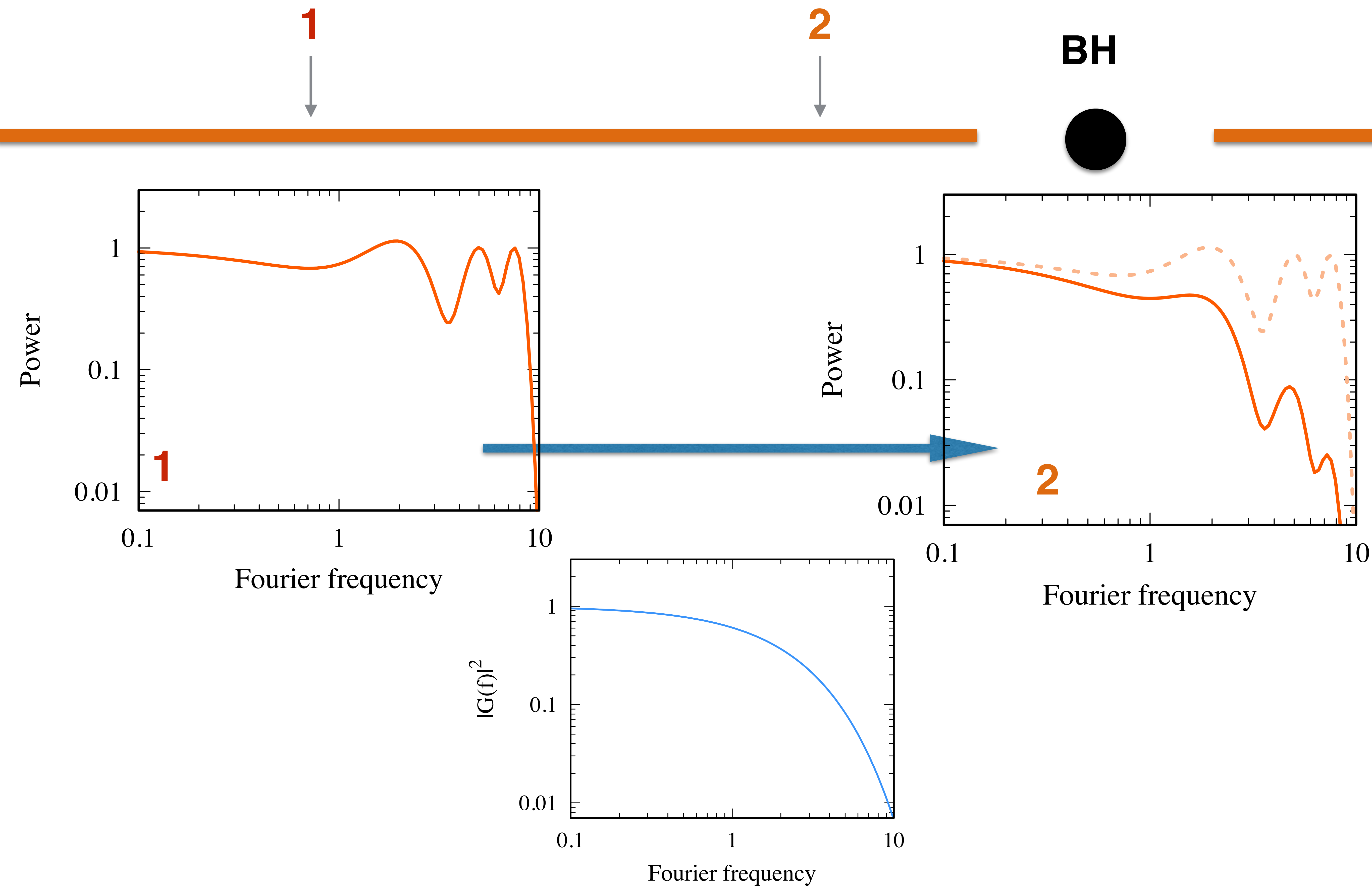
$$\dot{m}(R,t) = \int_{R_{\rm in}}^{R_{\rm out}} \mathrm{d}R' \, G_{\dot{M}}(R,R',t) \otimes_t \frac{a(R',t)}{R'}$$

Fourier transform

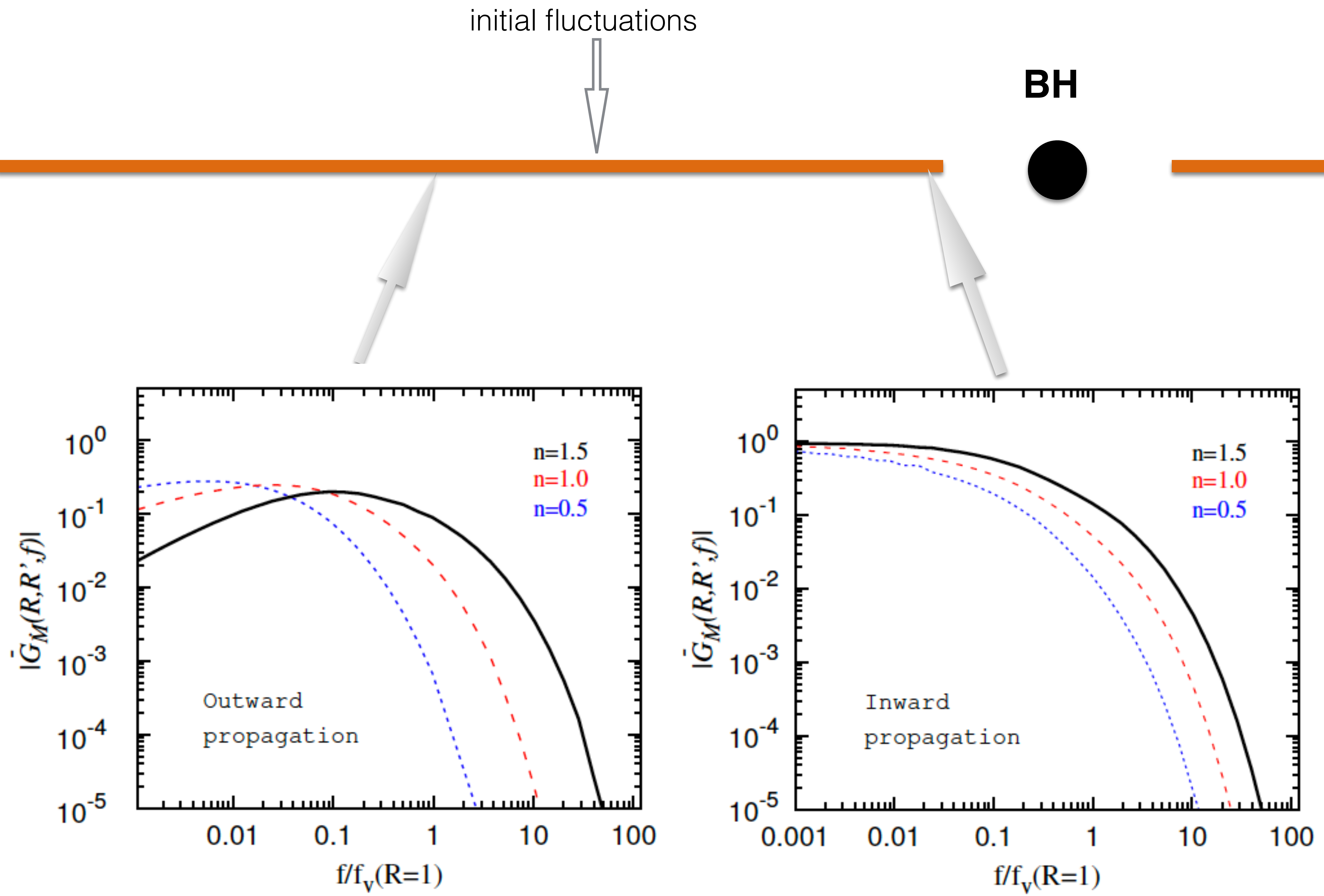
$$\overline{\dot{m}}(R,f) = \int_{R_{\rm in}}^{R_{\rm out}} \mathrm{d}R' \, \overline{G}_{\dot{M}}(R,R',f)\overline{A}(R',f)$$

We need to construct **Green's functions in the frequency domain**

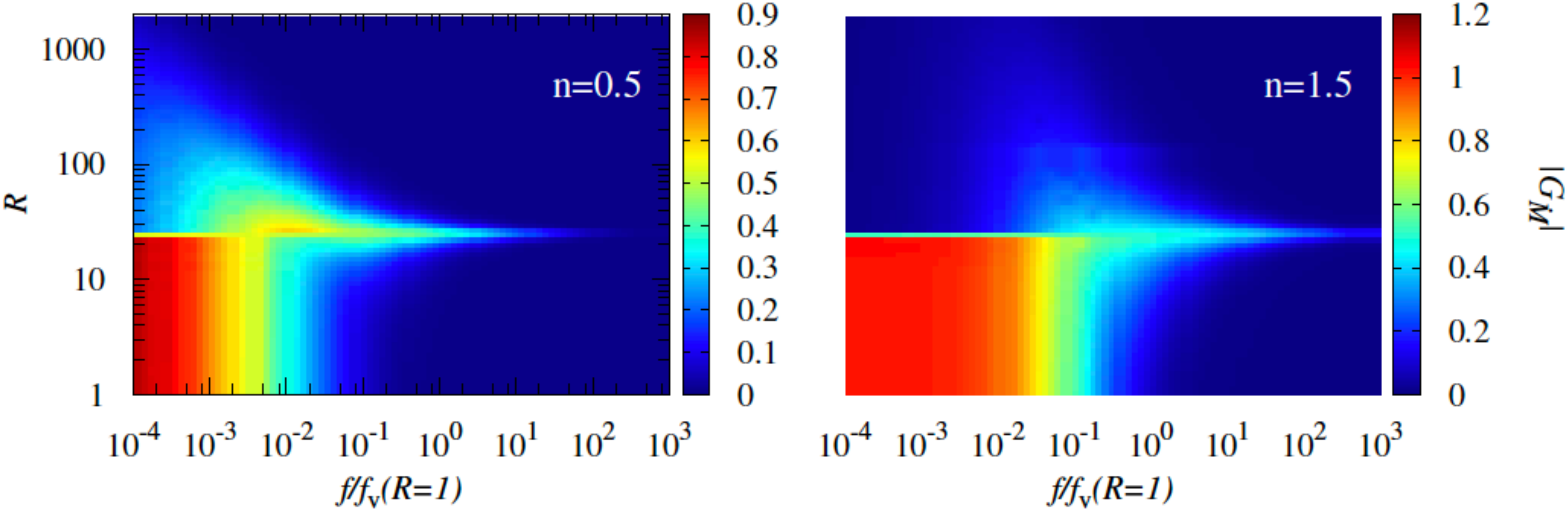
Mass accretion rate fluctuations in the time domain



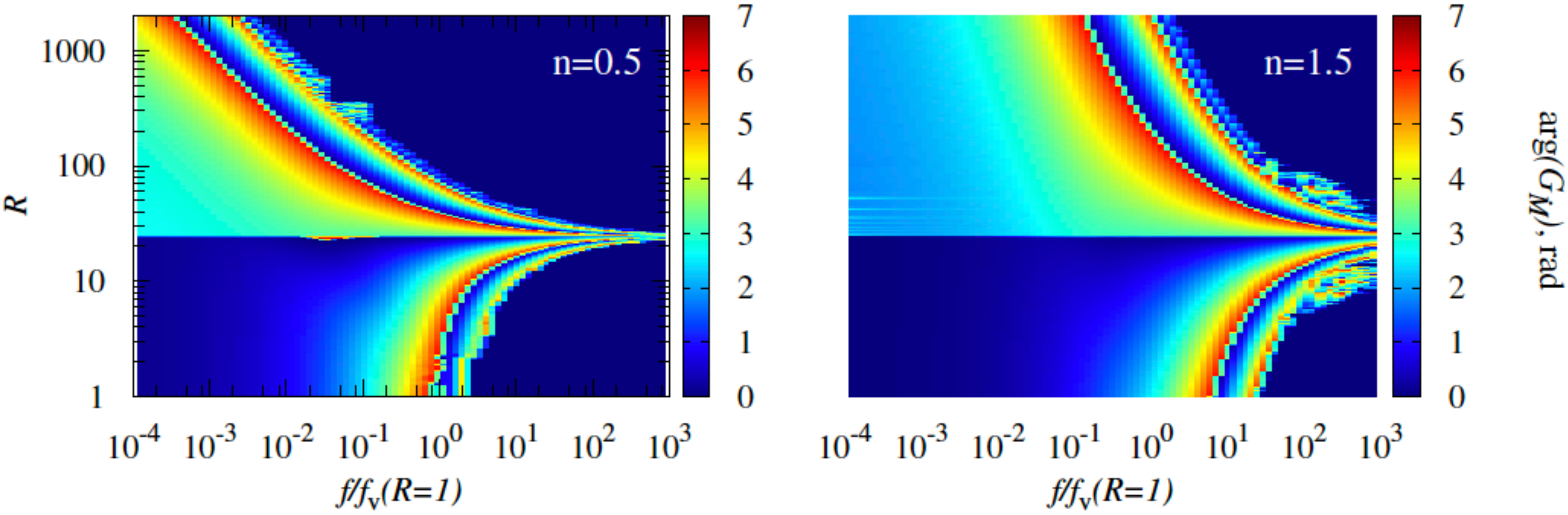
Mass accretion rate fluctuations in the frequency domain



The Green's function absolute value

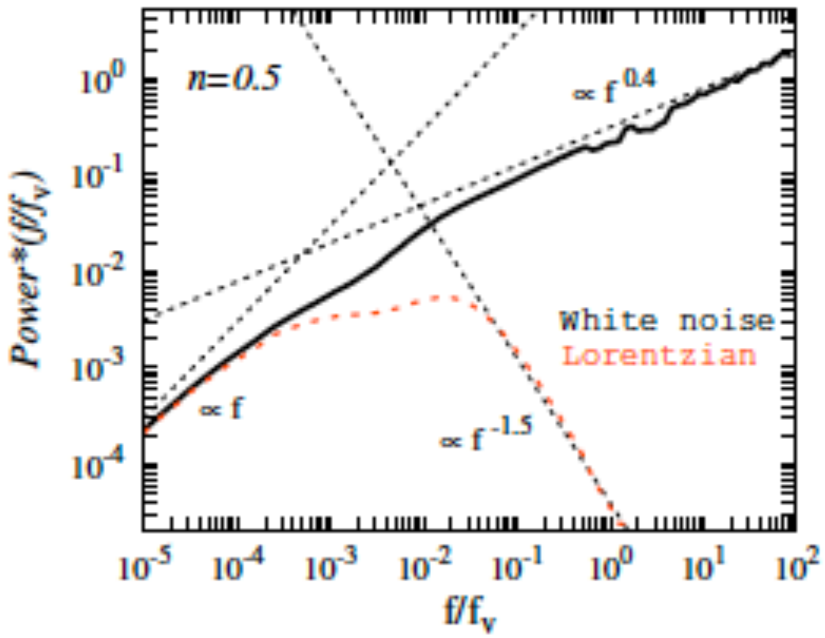


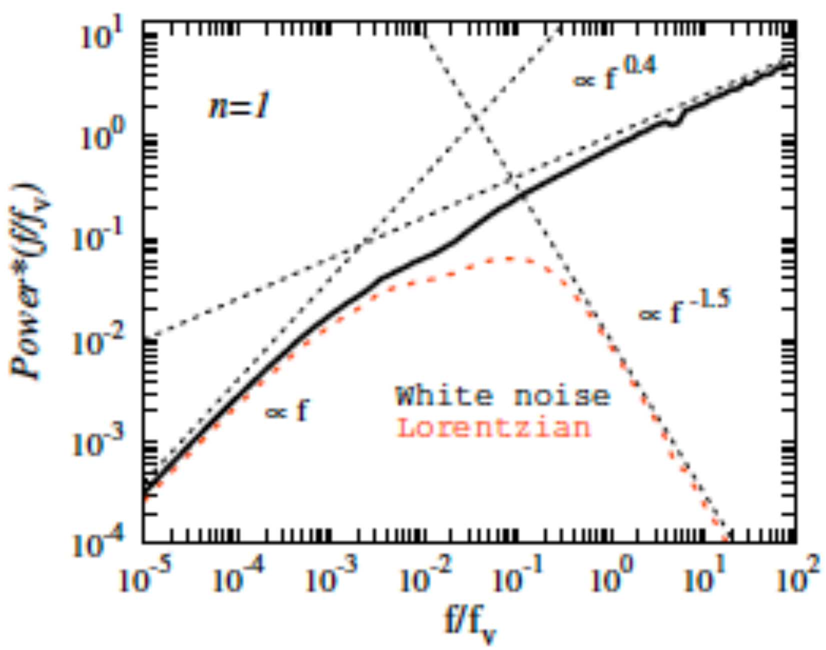
The Green's function phase angle

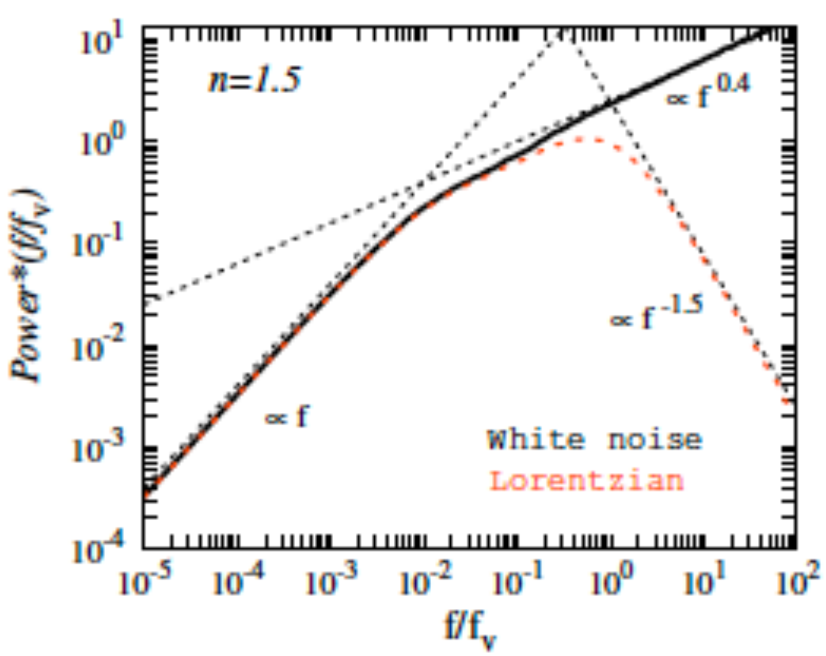


Power spectrum of the mass accretion rate

$$S_{\dot{m}}(R,f) = \int_{R_{\rm in}}^{R_{\rm out}} \frac{{\rm d}R'}{(R')^2} \Delta R(R') |\overline{G}(R,R',f)|^2 S_a(R',f)$$







The power of mass accretion rate **at R=50**.
Accretion disc is extended up to R=200.

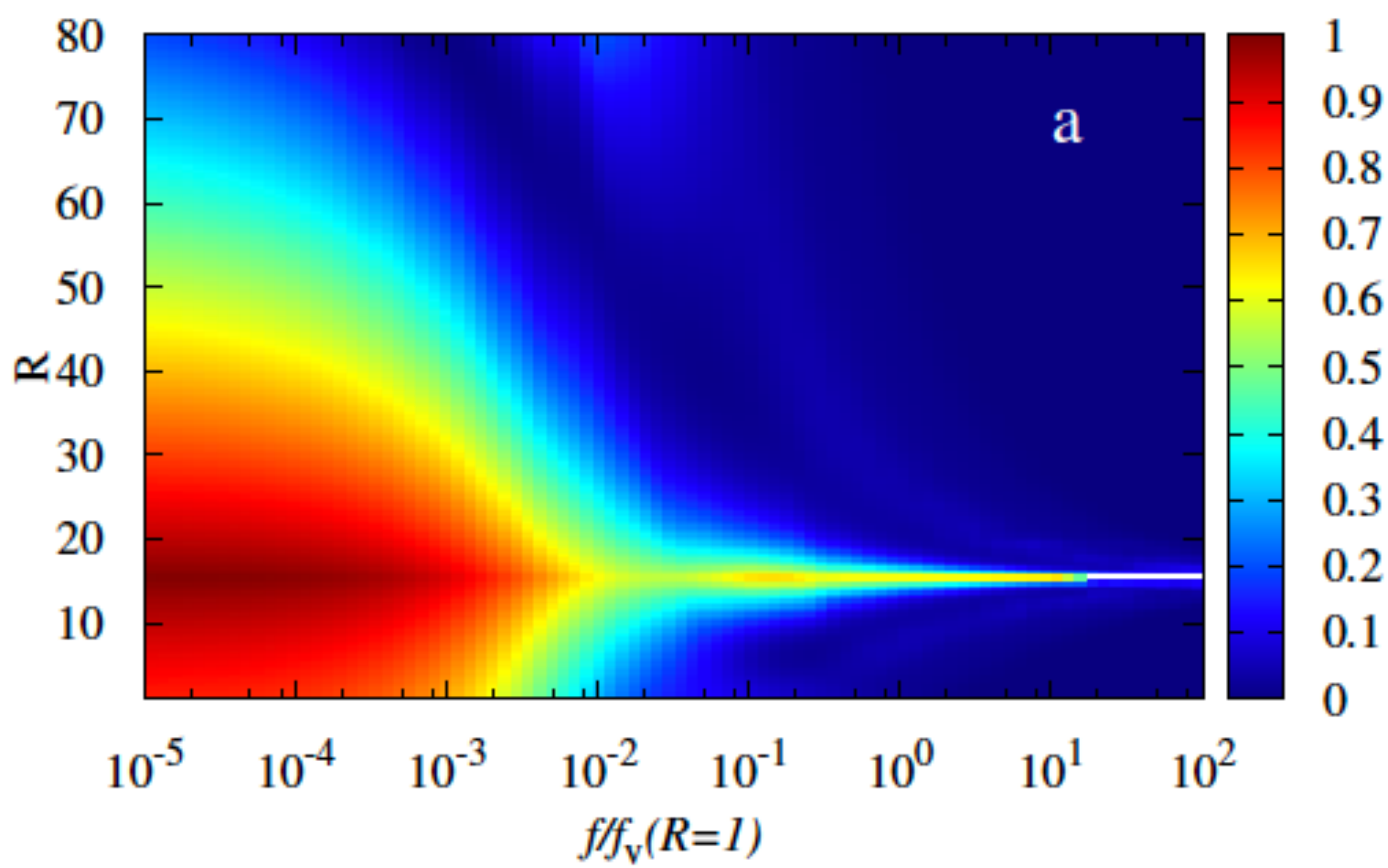
The power imprints the inner and outer radii
and
strongly depends on the power spectrum of initial fluctuations.

Zero-centred Lorentzian: $|a(f)|^2 \propto \frac{2}{\pi} \frac{\Delta f_R}{(\Delta f_R)^2 + f^2}$

Cross-correlation function of the mass accretion rate

$$C_{\dot{m}}(R_1, R_2\,|\,f) = \int_{R_{\rm in}}^{R_{\rm out}} \frac{{\rm d}R'}{(R')^2} \Delta R(R') \overline{G}(R_1, R', f) \overline{G}^*(R_2, R', f) \times S_a(R', f) \otimes \left[\delta(f) + \frac{S_{\dot{m}}(R', f)}{\dot{m}_0^2}\right]$$

Coherence function: $\mathrm{Coh}(R_1, R_2\,|\,f) \equiv \frac{|C_{\dot{m}}(R_1, R_2\,|\,f)|^2}{S_{\dot{m}}(R_1, f) S_{\dot{m}}(R_2, f)}$



Cross-correlation function of the mass accretion rate

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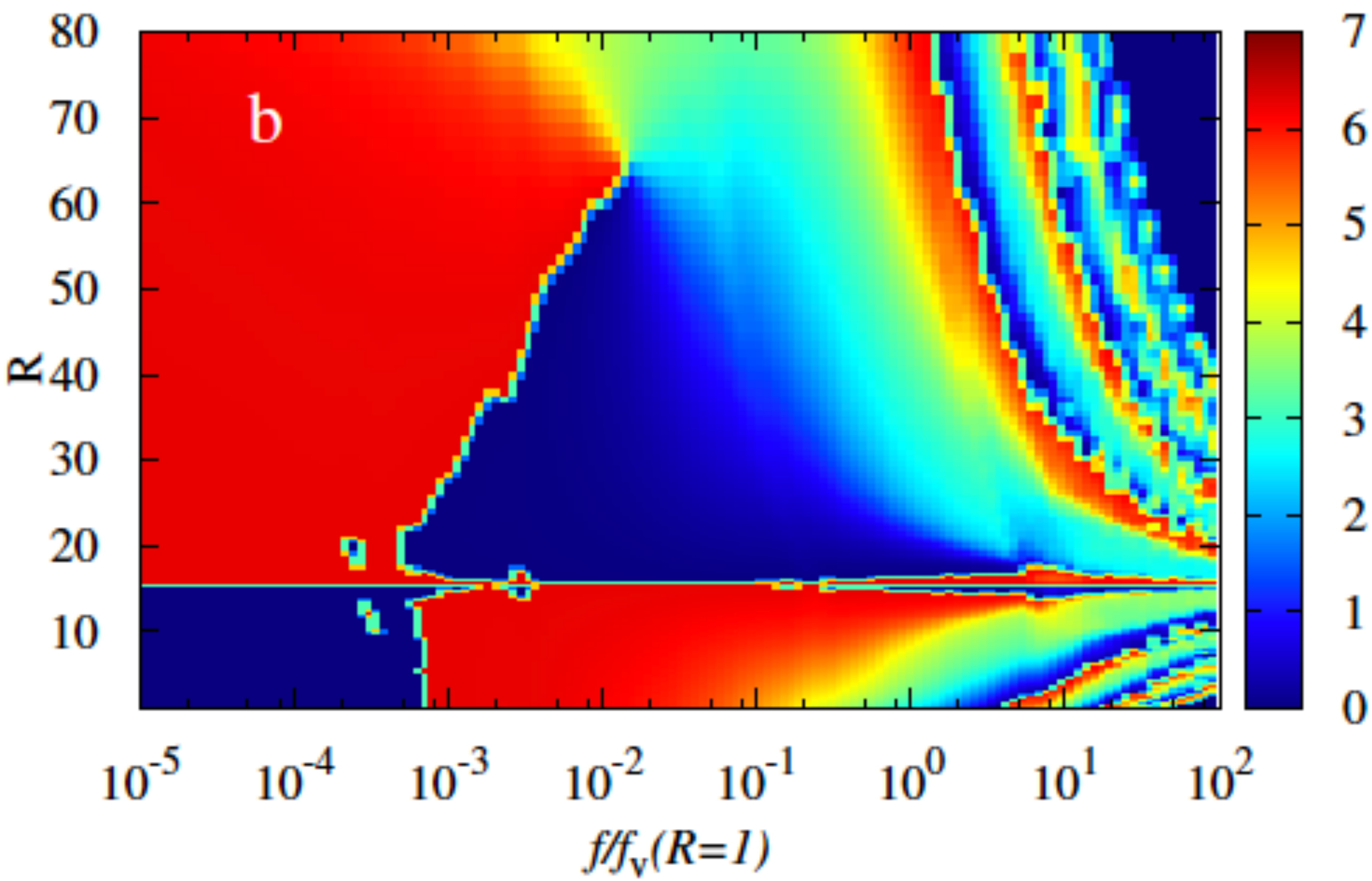
The phase angle of cross-correlation function



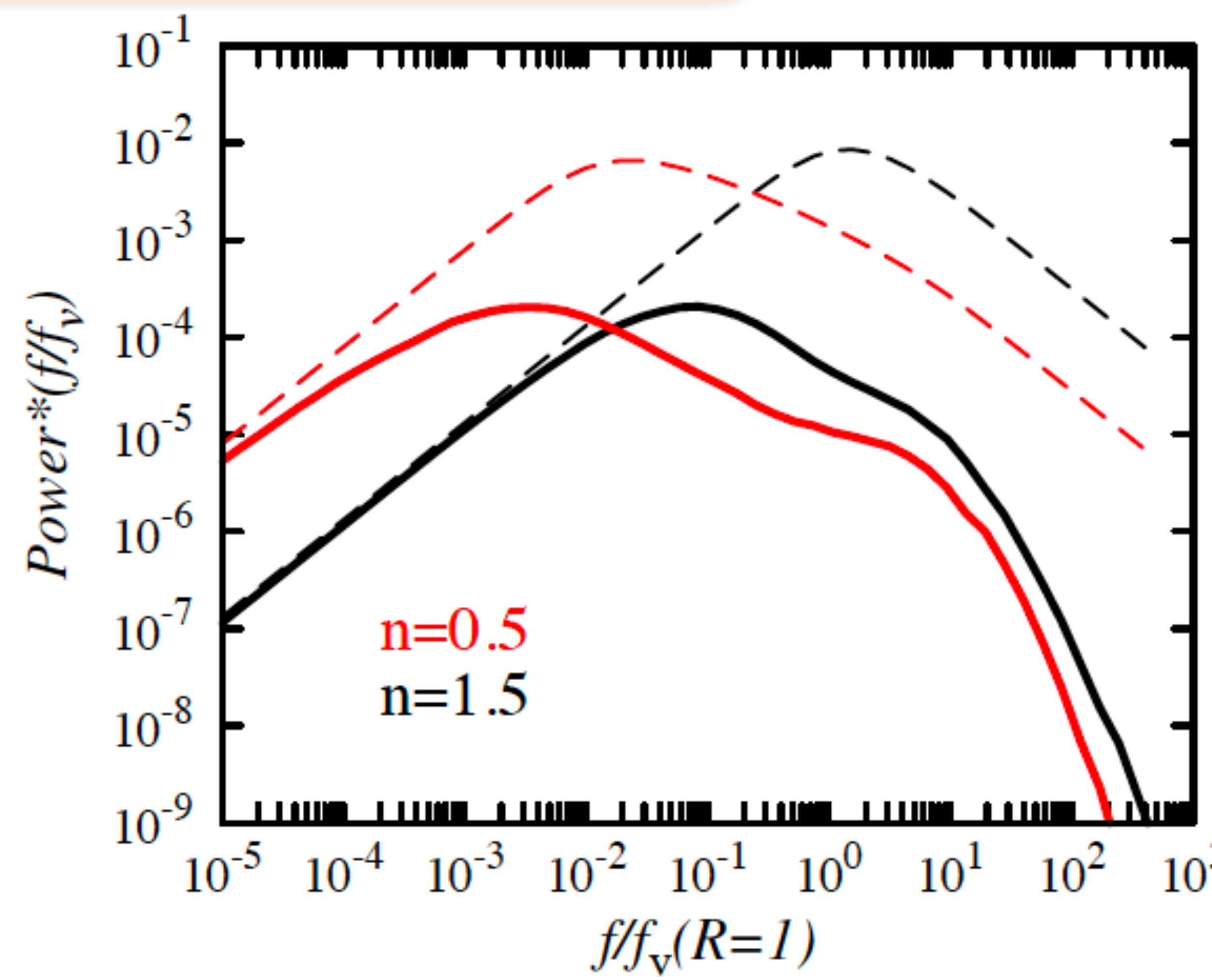
Power spectra of X-ray flux variability

$$F_{\rm sur} \propto R^{-3}$$

Flux at given energy band: $$f_{\rm h}(t) \simeq \int_{R_{\rm in}}^{R_{\rm out}} {\rm d}R' \, \frac{h(R')}{R'^2} \dot{m}(R', t)$$

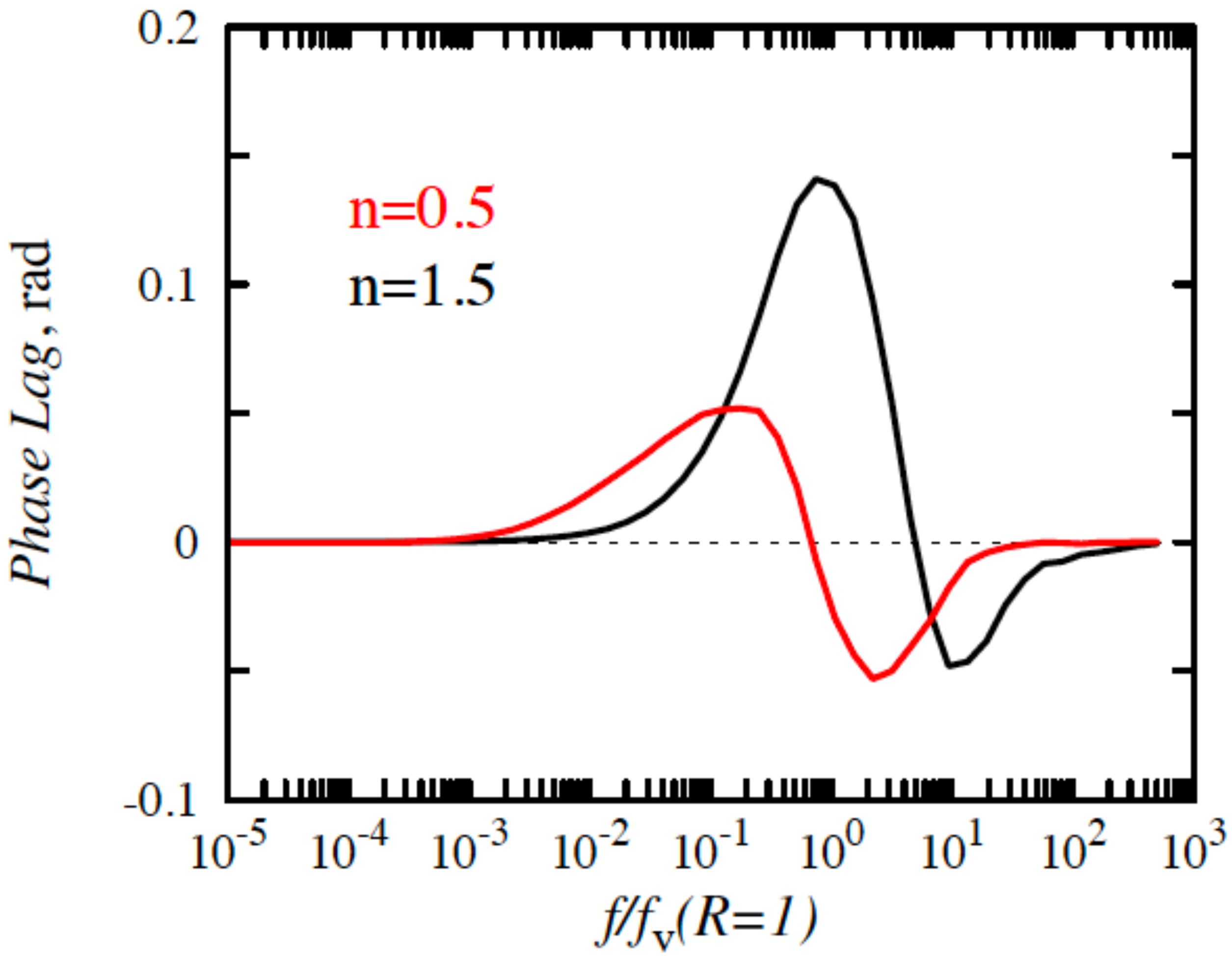
$$S_{f_{\rm h}}(f) = \int_{R_{\rm in}}^{R_{\rm out}} {\rm d}R_1 \int_{R_{\rm in}}^{R_{\rm out}} {\rm d}R_2 \, \frac{h(R_1)}{R_1^2} \frac{h(R_2)}{R_2^2} C_{\dot{m}}(R_1, R_2 \,|\, f)$$

$$h(R) = \begin{cases} 1, & \text{for } R < 3R_{\rm in} \\ \frac{1}{3}, & \text{for } R \geq 3R_{\rm in} \end{cases}$$

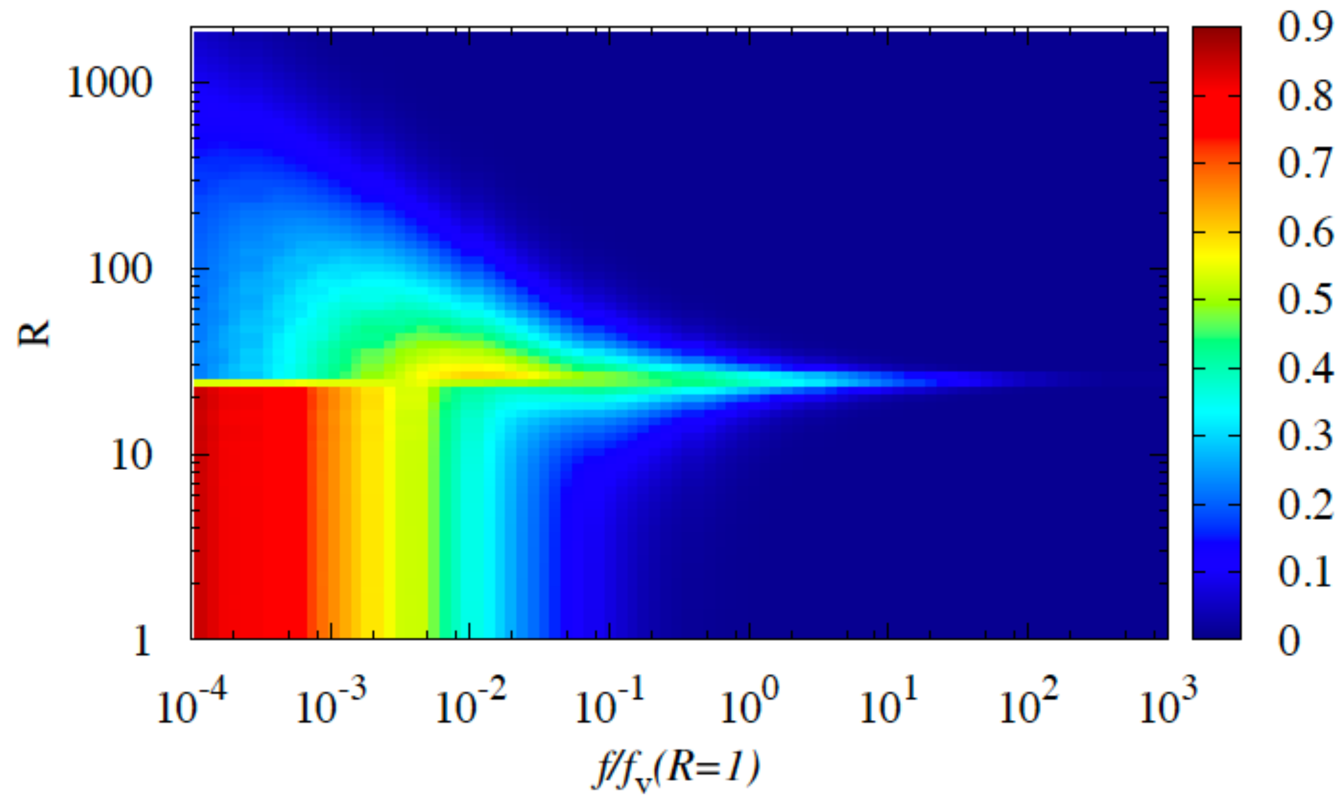


Cross-spectrum of X-ray flux variability

$$C_{\rm h,s}(f) = \int\limits_{R_{\rm in}}^{R_{\rm out}} {\rm d}R_1 \int\limits_{R_{\rm in}}^{R_{\rm out}} {\rm d}R_2 \, \frac{h(R_1)}{R_1^2} \frac{s(R_2)}{R_2^2} C_{\dot{m}}(R_1, R_2 \,|\, f)$$

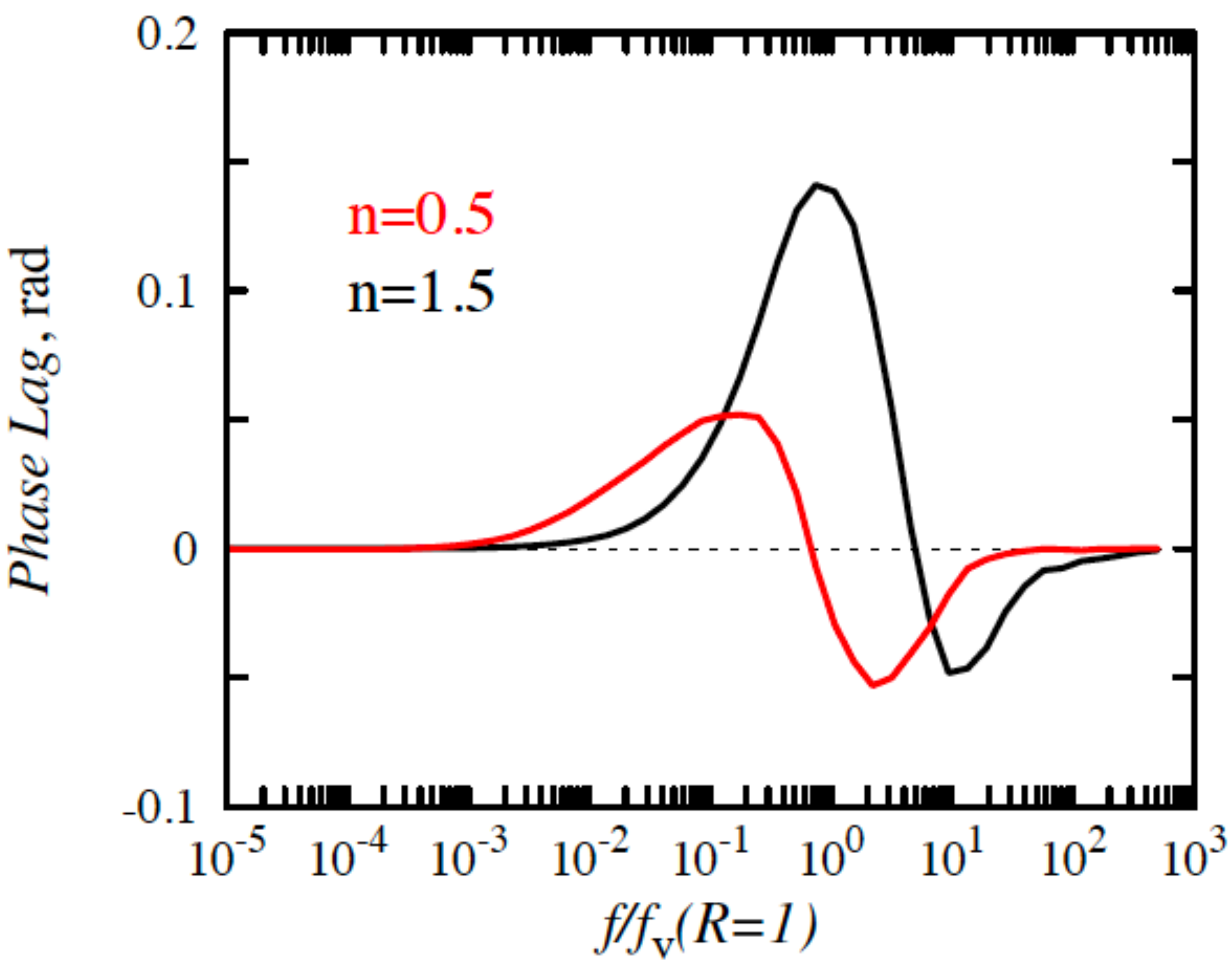


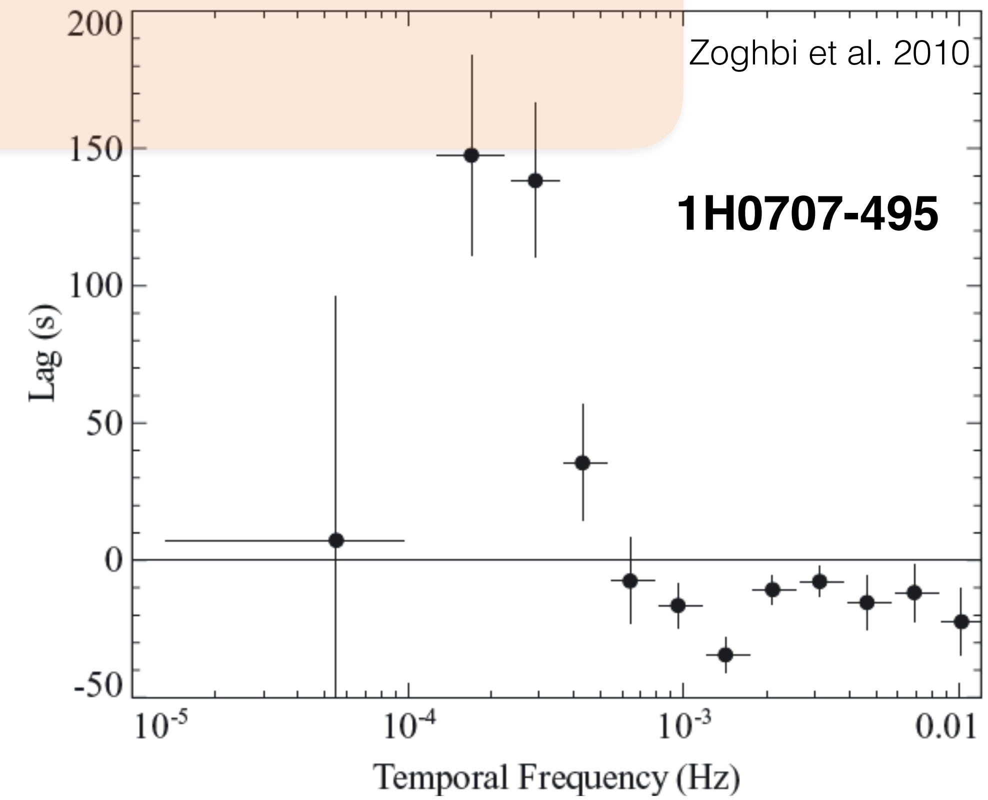
an example of Green's function in the frequency domain

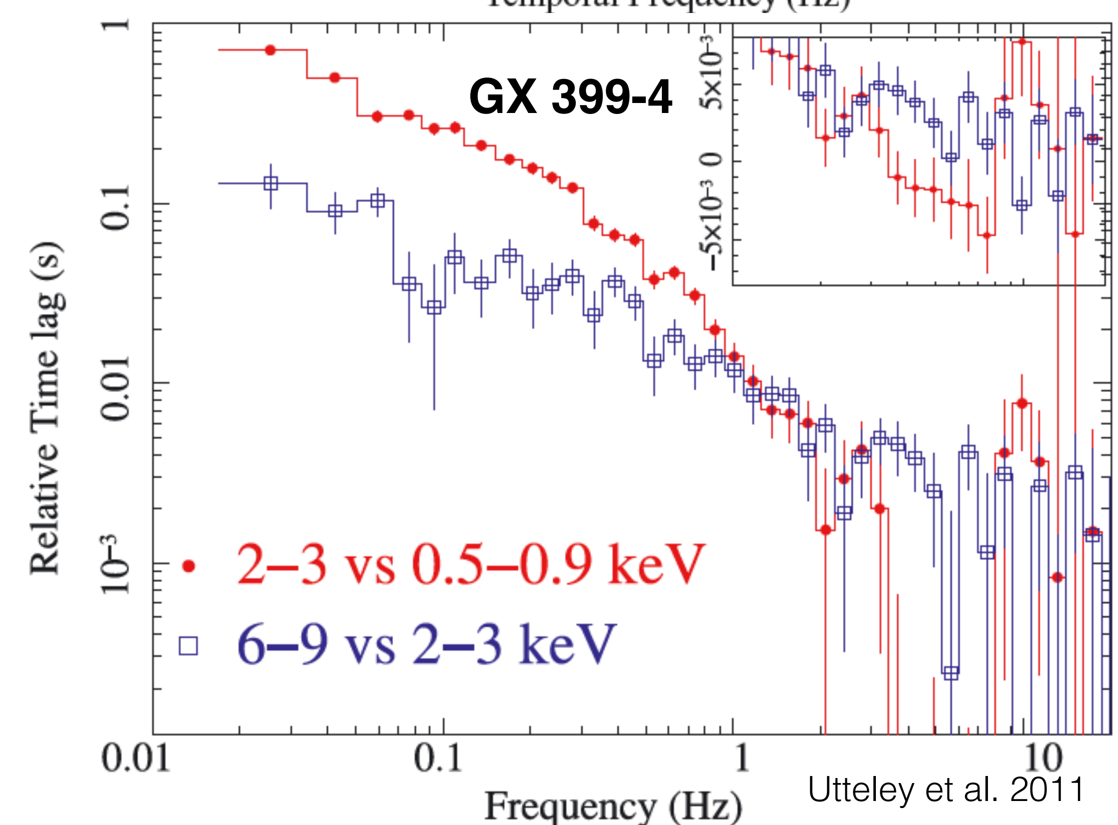


Cross-spectrum of X-ray flux variability

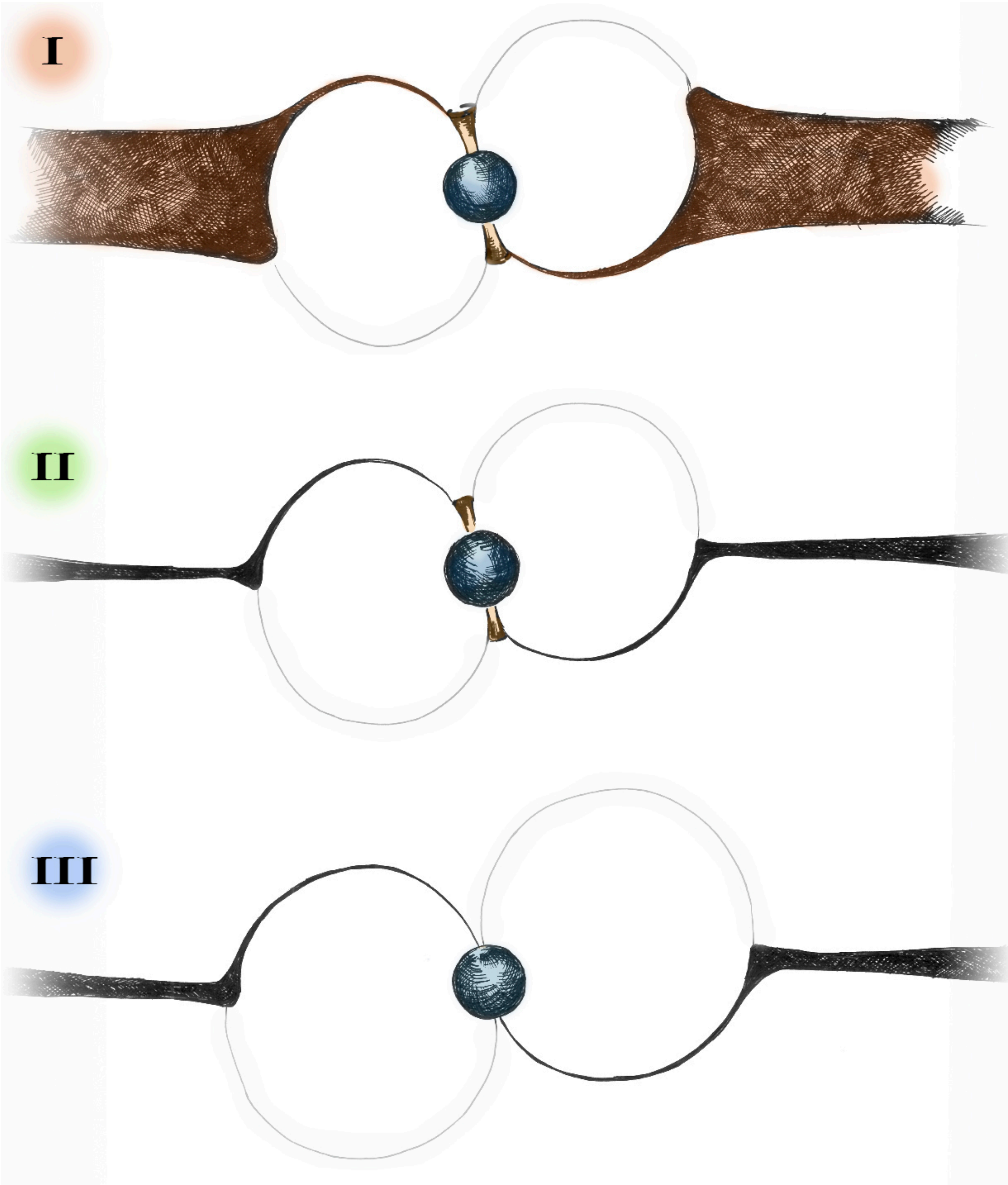
$$C_{\mathrm{h,s}}(f) = \int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} \mathrm{d}R_1 \int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} \mathrm{d}R_2 \, \frac{h(R_1)}{R_1^2} \frac{s(R_2)}{R_2^2} C_{\dot{m}}(R_1, R_2 \,|\, f)$$







Aperiodic variability in X-ray Pulsars

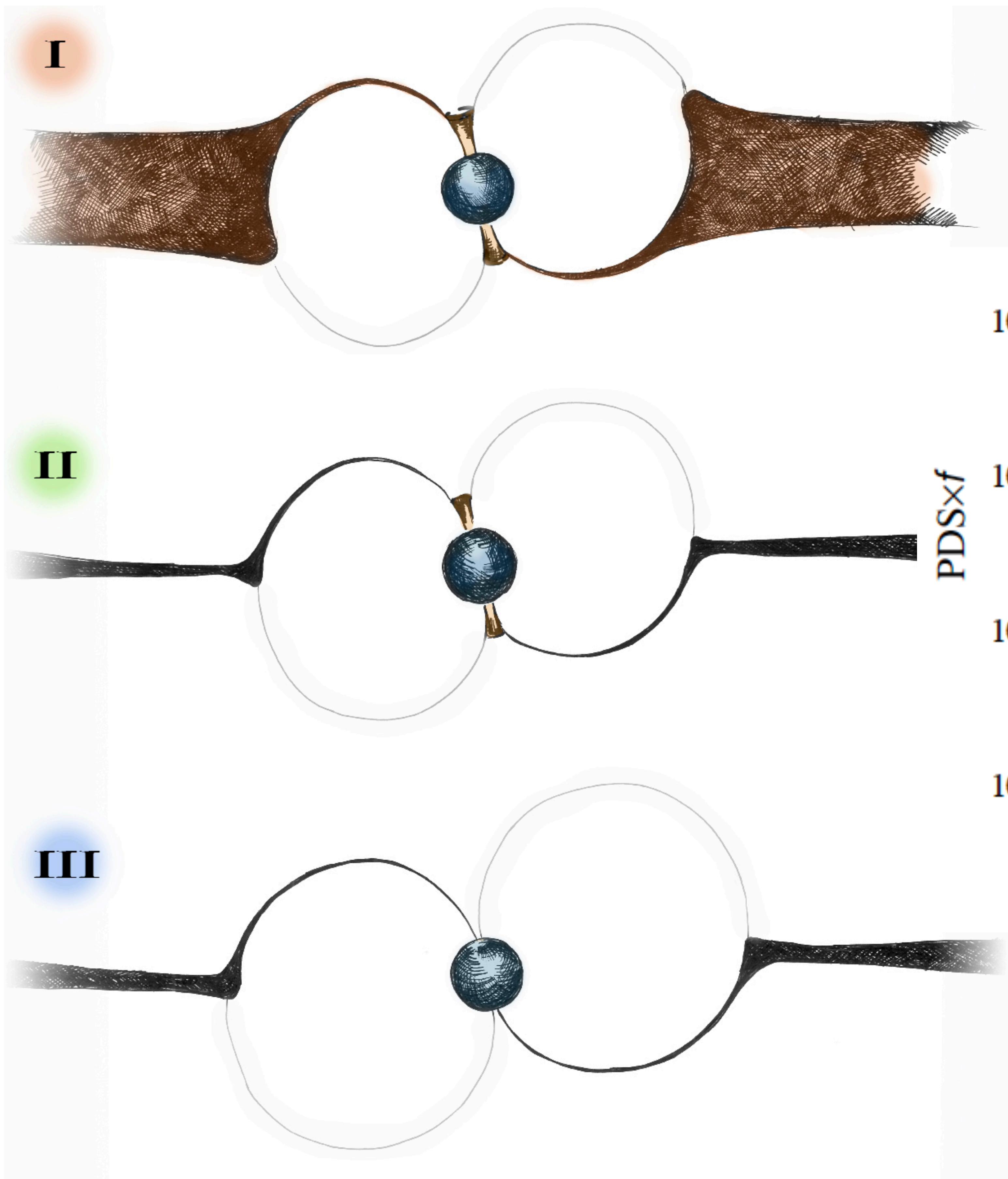


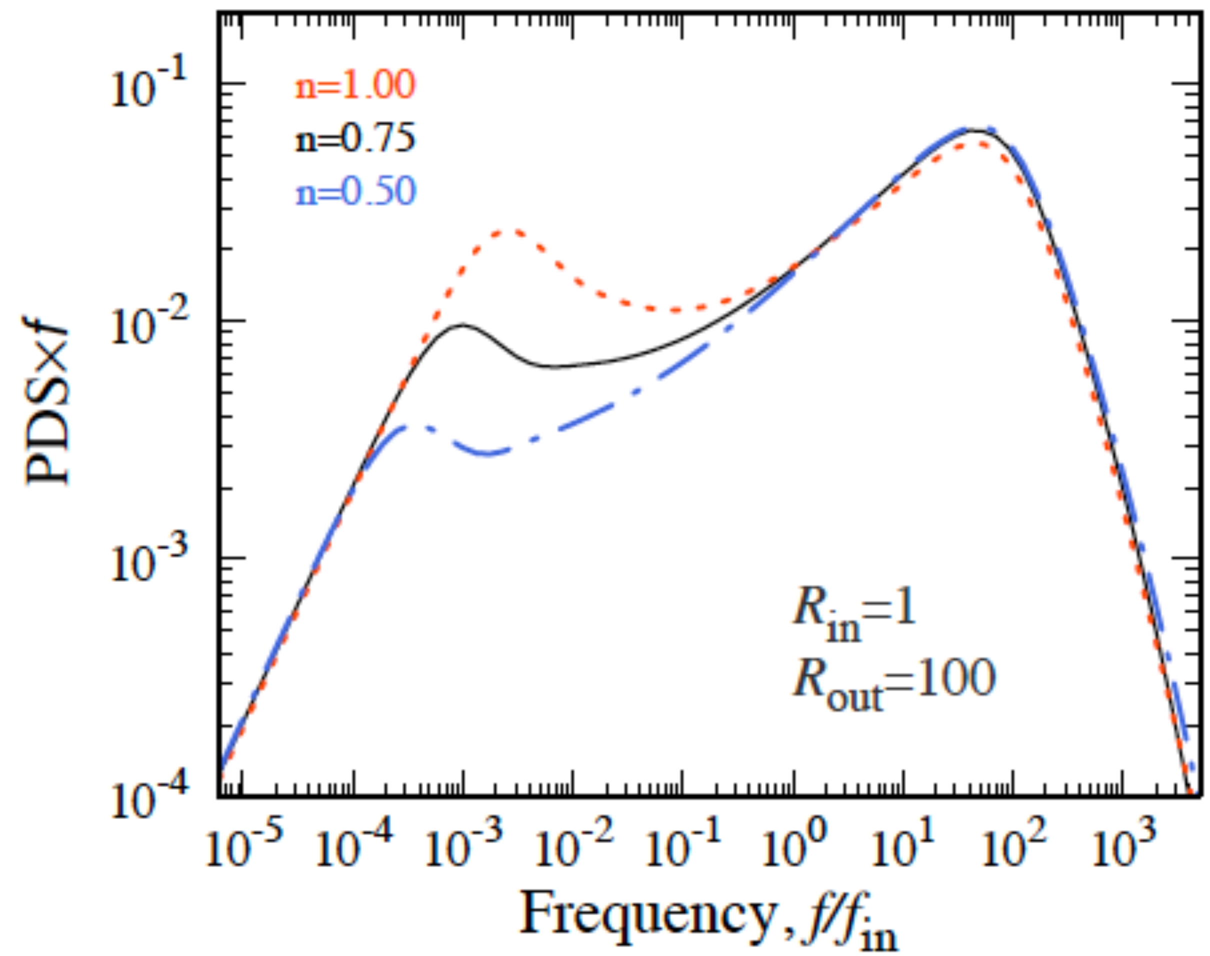
Accretion disc does not contribute significantly to X-ray energy flux

Mass accretion rate fluctuations at the NS surface replicate the fluctuations at the inner disc radius

Observed fluctuations of X-ray energy flux can be affected by variability in geometry of the emitting region

Aperiodic variability in X-ray Pulsars

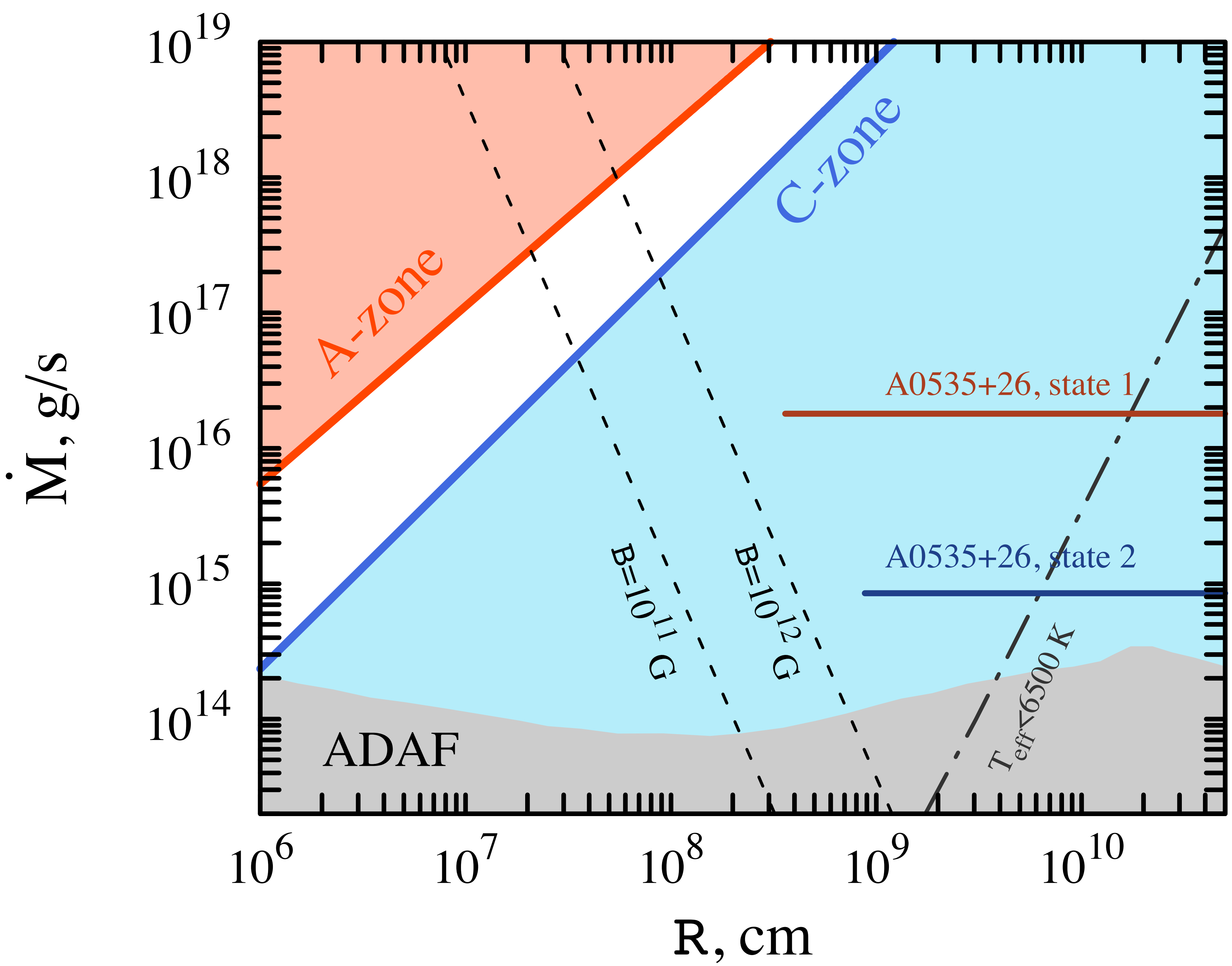




X-ray pulsar A0535+26

$E_{cyc,0}$=45 keV **$E_{cyc,1}$~100 keV** **P_{spin}~100 sec**

Two luminosity states: **$L_1=1.7*10^{35}$ erg/s** **$L_2=3.8*10^{36}$ erg/s**



Gas pressure dominated disc

The inner disc is still ionized

Hot spots on the surface ($L_{crit} > 10^{37}$ erg/s)

X-ray pulsar A0535+26

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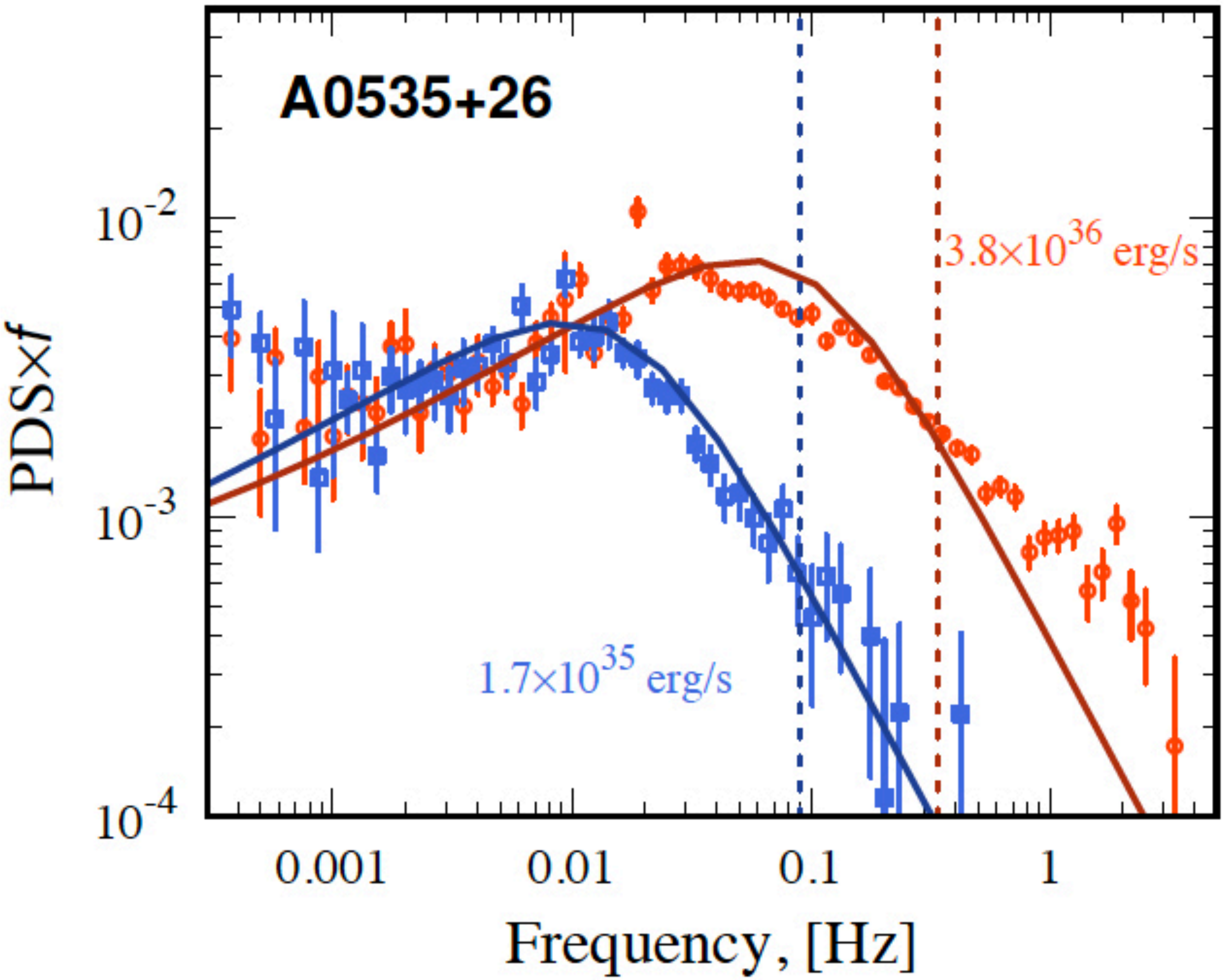

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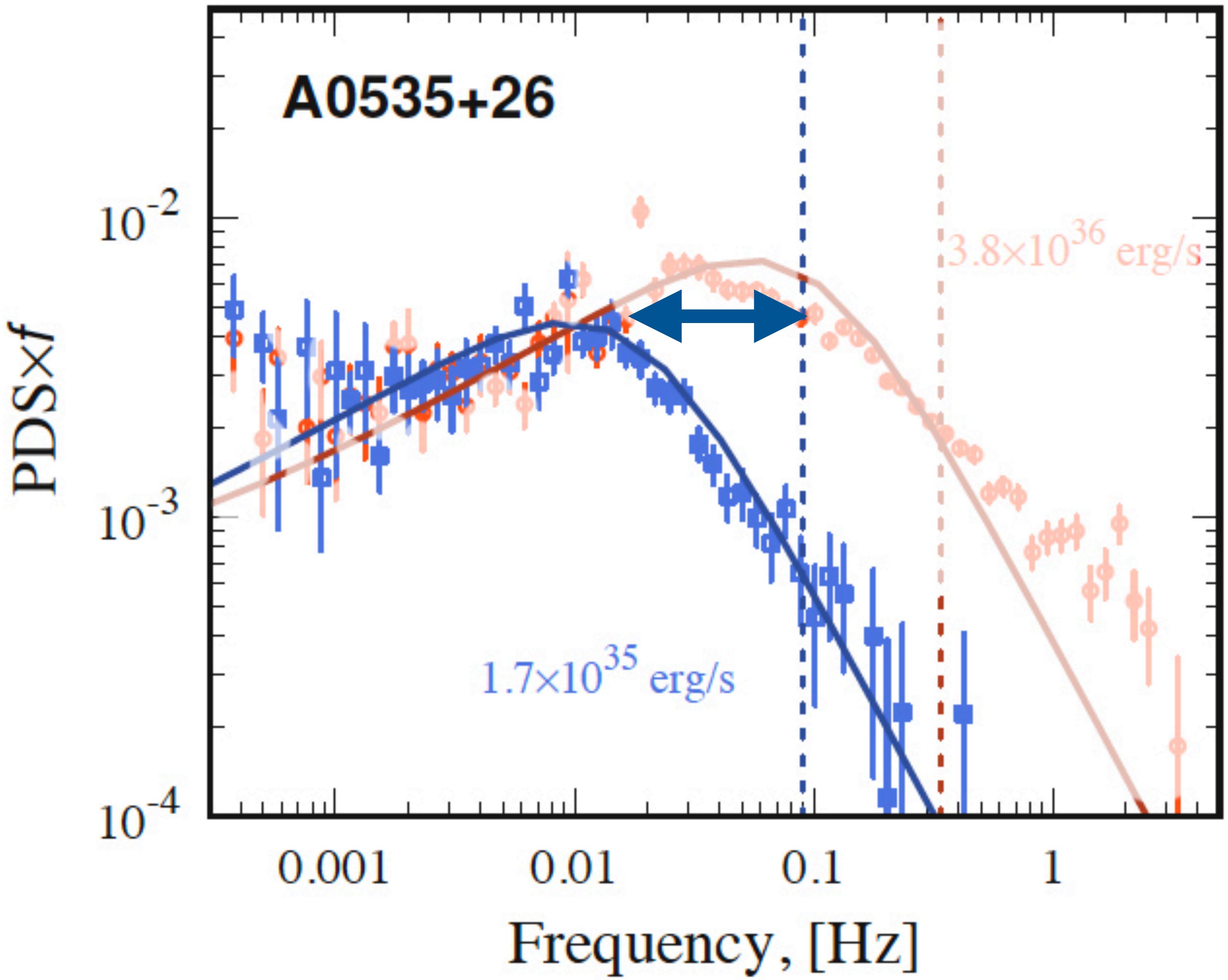

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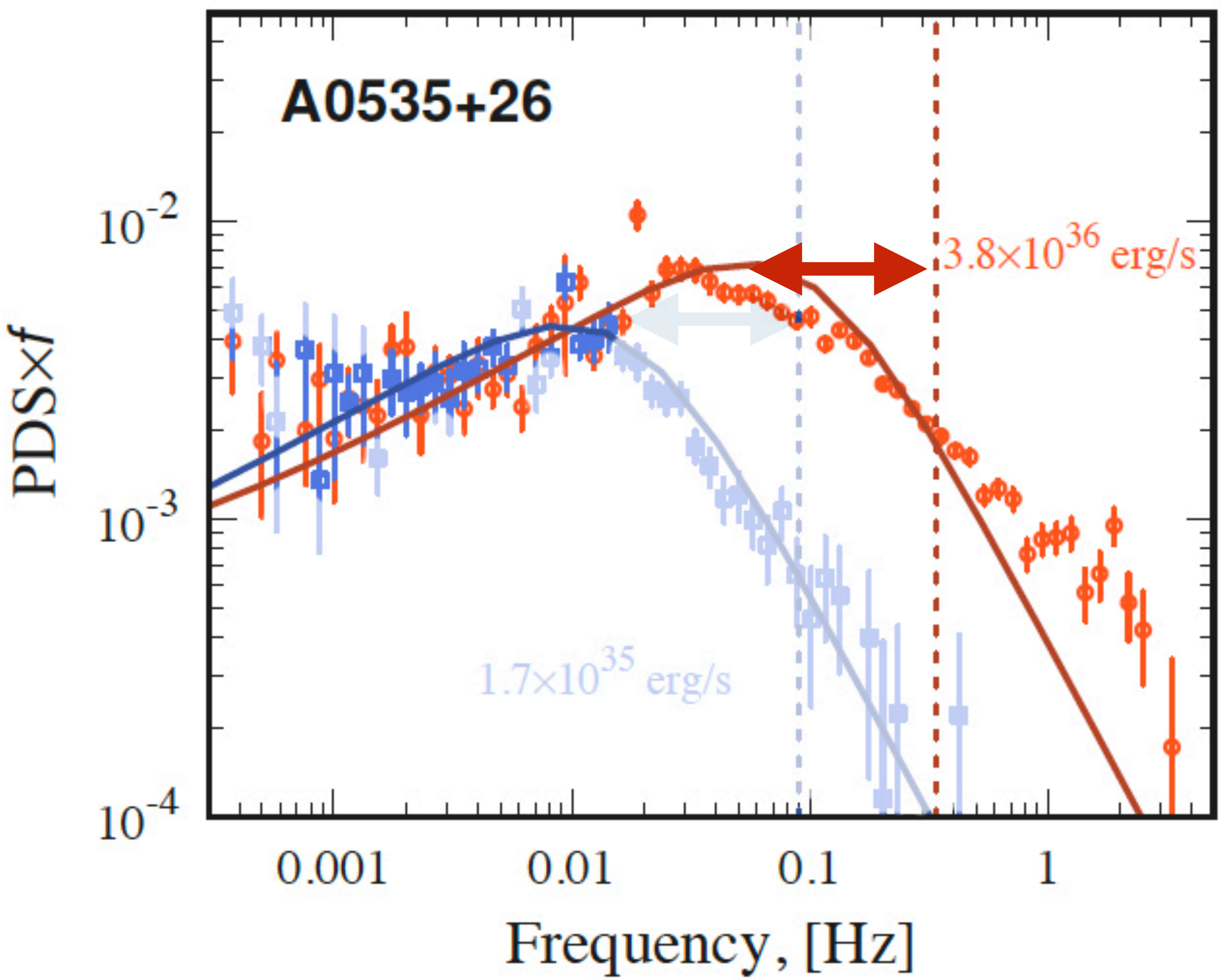
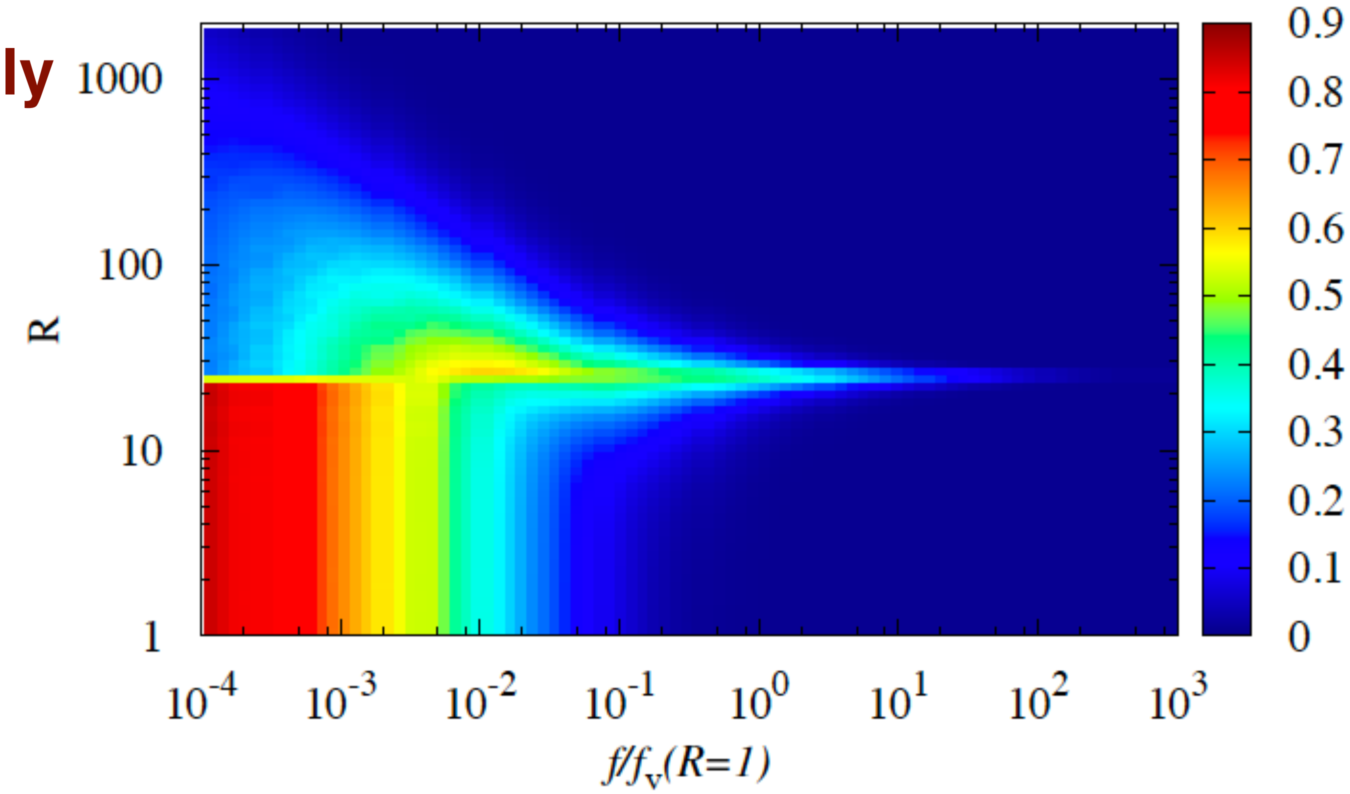
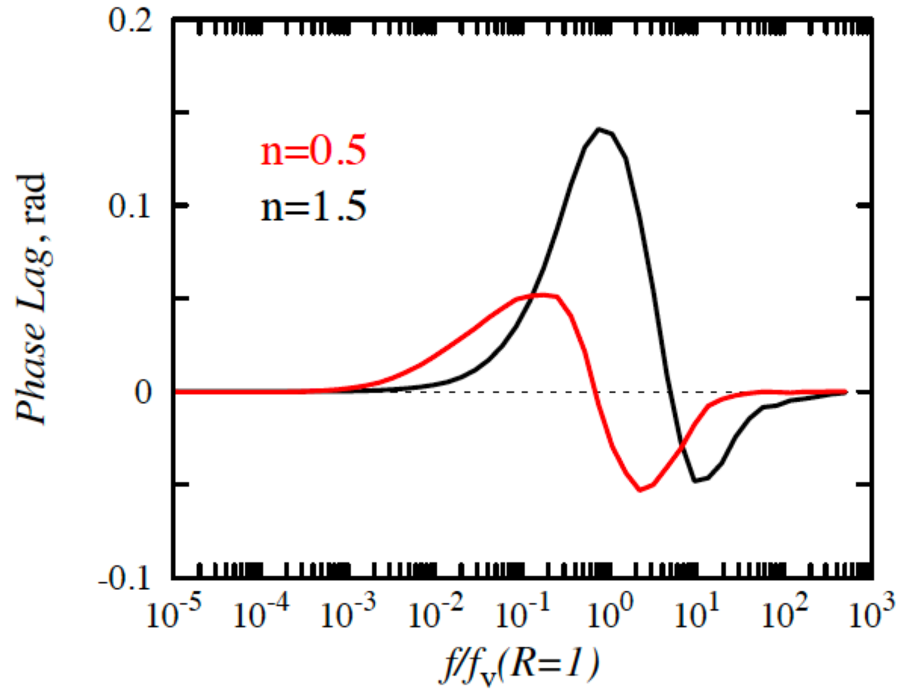


Conclusions

(a) viscous diffusion **suppresses effectively variability at time scales smaller than local viscous time**;

(b) the fluctuations of the mass accretion rate **propagate both inwards and outwards**;





(c) as a result, propagating fluctuations give rise not only to hard time lags as previously shown, but also **produce soft lags at high frequency** similar to those attributed to reflection reprocessing;

(d) **The break observed at high frequencies in the PDS of XRPs corresponds to the minimal time scale of the dynamo process** in a disc. As a result PDS of XRPs can be used as a method of independent measurements of magnetic field strength and structure in XRPs.